

Clemson University's Rotational Attendance Plan During COVID-19

Amy B. Gore,^{a*} Mary E. Kurz,^a Matthew J. Saltzman,^a Blake Splitter,^a William C. Bridges,^a
Neil J. Calkin^a

^aClemson University, Clemson, South Carolina 29634

*Corresponding author

Contact: aburto2@clemson.edu (ABG); mkurz@clemson.edu (MEK); mjs@clemson.edu (MJS);
bsplitt@clemson.edu (BS); wbrdgs@clemson.edu (WCB); calkin@clemson.edu (NJC)

Abstract. The COVID-19 pandemic forced universities to upend their class scheduling. At Clemson University, the administration implemented hybrid schedules for fall 2020, in which students attend classes partly online and partly in person. To limit exposures of COVID-19 in the classroom, we propose two rotational attendance models (the Three-Cohort Model and the Once-a-Week Model) that aim to allow in-person classroom time and minimize exposure between students. In a baseline strategy, students would interact with an average of 84 students per week and attend class in person 2.6 days a week. By contrast, the Three-Cohort Model and Once-a-Week Model achieve about 57 and 83 student interactions per week as well as 1.6 and 1.9 in-person student attendance days a week, respectively. Although these figures of merit may imply that the Three-Cohort Model is preferable, it achieves its results by forcing about 1,600 of the 21,000 students who want to attend courses in person to participate online instead, and forcing courses that meet twice a week to be attended twice in a three-week rotation. Considering the trade-offs between the figures of merit related to student interaction and anticipated implementation challenges, Clemson University implemented the Once-a-Week Model for fall 2020 and spring 2021.

Keywords: COVID-19; rotational attendance; integer programming; incidence graph; network problem

The COVID-19 pandemic caused significant disruption to many parts of the economy, including massive disturbances to the education workflow. Many educational institutions created policies to reduce large gatherings of students in classrooms and slow the spread of COVID-19. Different institutions used different approaches to reduce large gatherings. Freeman et al. (2021) reported that of the 50 largest public and 50 largest private institutions of higher learning (100 total), 66 used an approach with reduced numbers of students on campus, and 57 used an approach with reduced classroom density. Clemson University eventually used both approaches in its response to the COVID-19 pandemic.

Clemson's COVID-19 Strategy

Clemson University is a moderate-sized, public, doctoral-granting R1 institution in South Carolina, USA. According to the Clemson University Interactive Factbook, the university enrolled 20,868 undergraduate students (20,021 full time) and 5,538 graduate students (2,949 full time) in fall 2020. Undergraduate students were enrolled in an average of 15.00 credit hours per semester, whereas graduate students registered for an average of 7.84 credit hours (Clemson University 2020b).

In summer 2020, the Clemson University administration decided that maximizing in-person teaching was desirable and began their fall 2020 COVID-19 strategy with a reduced classroom capacity approach. Clemson University Facilities determined the capacity of each of the 483 teaching spaces such that students had six-foot separation. Contractors physically reconfigured most teaching spaces by removing seats to enforce social distancing. As seen in Figure 1, only seats centered inside each six-foot circle were retained. Nontraditional spaces such as basketball courts at the recreation center were also used to create new classrooms. Clemson Computing and Information Technology ensured that each classroom had appropriate video cameras and speakers to facilitate any online instruction components, resulting in 381 updated classrooms (Clemson University 2020a).

The Registrar's Office reconfigured the fall 2020 course schedule using the new rooms and capacities, respecting the enrollments of existing students. Unfortunately, even after moving some classes online and allowing voluntary online attendance, available classroom capacity was insufficient to accommodate a classroom density reduction approach. The next step in Clemson University's COVID-19 response used changes to course modality to reduce the number of students on campus on a given day.

For fall 2020, all class instruction (undergraduate and graduate) could be conducted using one of two modalities: online (synchronous or asynchronous) or hybrid. Hybrid classes were designed as a combination of online and in-person instruction. The hybrid courses were intended to reduce the number of students on campus on a given day and allow for proper social distancing in classrooms. Specific details of the hybrid class approach follow.

For each day that a hybrid meeting was conducted, a subset of students (less than or equal to the safe social-distancing capacity of the room) would be selected for in-person instruction in the classroom. The complement of that subset would receive online instruction during that meeting. At the next meeting of the hybrid course, another subset

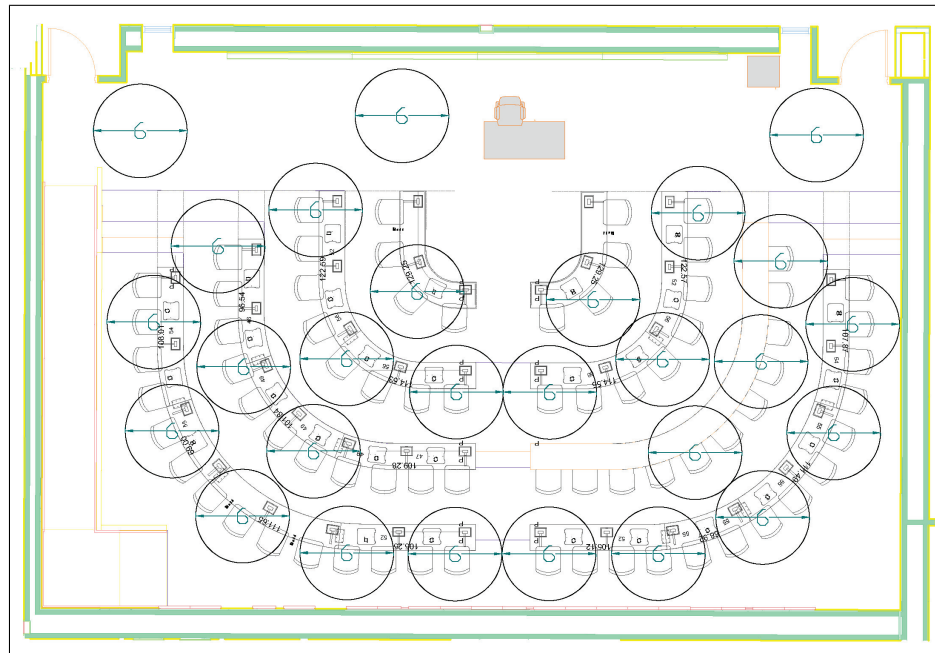


Figure 1. (Color online) This Chart Displays a Seating Layout for Social Distancing in a Seminar Room
Source. Garber (2021).

Note. Circles represent 6-foot-diameter bubbles.

of students would be selected for in-person instruction, and the complement would be online. This process was repeated for each course meeting, creating a rotating schedule. Courses were assigned to rooms such that each subset could fit into the rooms given their social-distancing capacity.

Whereas Freeman et al. (2021) report on the prevalence of hybrid instruction and classroom density reduction, they do not report how either strategy was implemented. University of Pittsburgh, University Center for Teaching and Learning (2021) indicated that faculty were empowered and expected to assign students to attendance days (called a “rotating cohort”) when a class’s enrollment would not fit into its assigned reduced-density classroom. Cornell University utilized a similar approach in spring 2021, called “rotational hybrid,” in which the instructor assigned students to in-person attendance days (Cornell University 2021).

Clemson University administration enlisted the authors to use mathematical optimization concepts to assign students to their hybrid course in-person attendance days. Spears School of Business at Oklahoma State University also built a mathematical model; however, their goal was to determine classroom assignments for courses to maximize face-to-face

offerings (Johnson and Wilson 2021). Their approach also differs in that the model did not include all classes at the university. Researchers at Georgia Tech did study a similar problem over their entire university (Navabi-Shirazi et al. 2021). Their work differs in that it focused on selecting course modality, an input in our model. Cornell University also made use of mathematical modeling in response to COVID-19 but chose to focus their research on testing strategy as opposed to course scheduling (Frazier et al. 2022). To the best of our knowledge, Clemson’s approach, both being centralized and utilizing rotational attendance (“rotational” because assignment groups cycle each week), is unique. We show here that our approach provides measurable advantages over some initial assignment policies and strategies first considered at Clemson.

The subsets of students that attend each in-person course can be chosen in several ways. At Clemson University, faculty of record were able to choose the course modality. Students could choose to participate in some or all of their classes online (regardless of the course modality), so the total number of students attending hybrid classes was significantly smaller than the total number of students registered. Therefore, we considered several policies, such as allowing the faculty to create the subsets, allowing the students to choose their day to meet in person, and making random assignments of subsets. Even with the reduced number of hybrid-eligible students, the goals of allowing for social distancing in classrooms and reducing the interaction of students on campus were not easily accomplished with these proposed policies. We quickly understood that the problem of limiting and coordinating the number of student interactions while respecting the required social distancing for the hybrid classes was amenable to an optimization approach.

At a bare minimum, the constraints imposed on instruction were that only a fixed number of students could be in each room and that each student should be able to attend each in-person class once a week. In such a system, each hybrid class’s daily attendance can be modeled as a random solution by implementing a default “round-robin” approach. With this strategy, each course’s students would be assigned to an attendance day: if a course met three days a week, students 1, 4, and 7 (etc.) would be assigned to day 1; students 2, 5, and 8 (etc.) to day 2; and students 3, 6, and 9 (etc.) to day 3. This method has the advantage of being easily implemented but provides no coordination or global view of the decisions being made. As such, it was never seriously considered for implementation at Clemson, but it was used as a baseline to assess the efficacy of our models.

Next, we evaluated timetabling as a possible approach to the university’s problem. A timetable is defined as a “schedule of meetings of classes (or students) and teachers over a definite period of time requiring certain resources” (Junginger 1986). The traditional approach to timetabling is to use a mathematical program to at least semiautomatically assign courses to set meeting times. Many timetabling problems also take on the objectives of creating assignments of instructors to courses, courses to classrooms, and/or students to courses. Even so, the environment in which our problem is solved does intersect with many previous timetabling works. The work of Stallaert (1997) uses a two-part approach with an integer program and a heuristic to solve the timetabling problem but does not consider student assignments. Similar to Hinkin and Thompson’s (2002) approach to eliminate conflicts created by a system in which professors made localized scheduling decisions, our model was proactively formed to prevent the problems of a localized approach—although Hinkin and Thompson’s decisions also include a larger scope, including course time and location. The integer program created by Martin (2004) only considers instructor assignment to courses, classrooms, and times, with no consideration of student scheduling. The work by Ogris et al. (2016) includes prioritizing multiple soft constraints, a topic discussed by the team in great detail but unfortunately not pursued because of the tight deadlines to which this research was subject. Gonzalez et al. (2018) assigned students to courses but also determined how many sections would be offered and which faculty would be assigned to the sections—a much larger scope than our setting. Overall, our problem differs from timetabling in that the normal decision variables of timetabling were given as parameters and constraints in this situation. We were not able to change the schedule or modality for any classes or alter the sections of courses in which students are enrolled. Our objective is instead to place students into attendance groups based on these predefined inputs.

In this paper, we analyze two optimization models that we developed to achieve the goal of minimizing the number of student interactions while respecting social-distancing classroom capacities. We developed the two models in sequence based on the way the university administration revealed their goals to us. The first goal provided was minimizing student interactions. This objective resulted in the Three-Cohort Model (3CM). In 3CM, students are partitioned into three groups, or cohorts, so that on each class day, one cohort would be on campus with in-person class(es) while the other two cohorts would stay off

campus and partake in classes online. This strategy worked well for traditional three-day-a-week classes, but for classes that did not meet three times, students might have different numbers of total in-person class meetings for the semester. At this point, the administration issued a second set of goals—specifically, that every student be able to attend hybrid classes exactly once per week per course. This requirement resulted in the Once-a-Week Model (OWM), which was implemented by the university. After providing details of these models, we explain our results and analyze how other, decentralized decision-making approaches can result in observably worse outcomes.

Terminology and Available Data

The following terminology is used throughout this paper.

- A *course* is a cohesive offering of lectures and educational experiences offered at a particular set of times and location(s) with instruction delivered by one or more instructors. A simple example of a course is MATH 101 meeting Monday, Wednesday, and Friday at 10:10 a.m. in Math Building Room 103. Each student registers for a collection of courses for a semester. For our purposes, students' registration status does not change over the term.
- A *meeting* is a single gathering of a course. Every course has at least one meeting per week. We often refer to meetings based on which day in a week they occur. The class MATH 101 has three meetings per week on Monday, Wednesday, and Friday.
- Two students *interact* if they are both present for the same in-person meeting of a course.

Additional notation and terminology specific to the models are introduced in the appropriate sections.

Clemson University provided the following data, which we used for the fall 2020 rotational assignment (Wardlaw 2020, Griffin 2020):

- 2,008 students opting for online-only instruction and
- 77,903 rows of pertinent hybrid registration information for
 - 21,007 hybrid students and
 - 3,059 unique hybrid courses.

Clemson University also provided fall 2019 data, which we used in prototyping the models described here.

Clemson University's Implementation of the Rotational Approach

In contrast to the University of Pittsburgh and Cornell practices, the Clemson University administration determined that allowing instructors to assign students to attendance days would be impractical. Specific reasons included (1) some instructors did not want the responsibility of making these decisions; (2) some instructors were concerned about how to communicate this information to students; (3) some instructors were concerned about complaints from students based on the decisions made by their other instructors; and (4) some administrators were concerned about how such distributed decision making would impact on-campus student interaction. Specifically, allowing students to choose their attendance days could result in widely varying number of students on different in-person attendance days (i.e., smaller Friday attendance) and reduce the quality of the students' and the instructor's experience.

Fortunately, while Academic Affairs struggled with these issues, Housing and Dining determined that students with meal plans would be assigned to meal times in the dining halls to ensure adequate social distancing. To communicate these assignments to the students, Clemson Computing and Information Technology developed an app that was used to inform students about their meal assignments as well as their daily attendance mode for each class they were enrolled in. The authors provided the daily attendance assignment based on the solution to the optimization model. This information was provided to the students prior to the start of the semester. See the Discussion section for a sample student view of the app. The authors were not involved in the development of the app.

Three-Cohort Model

One strategy for limiting student interactions is to confine students to groups and prevent interactions between groups. The 3CM creates a rotational attendance pattern of exactly three groups. The model requires that all students attending hybrid classes be assigned to one of three distinct groups, called "cohorts," such that no cohort is scheduled to be on campus simultaneously with any other cohort. Thus, any student's interactions are limited to other students within their cohort. On each weekday, one cohort is assigned to attend classes in person while the other two attend classes online. For example, any Monday-Wednesday-Friday course could easily partition their students into three cohorts, and every student would attend the class in person once per week on their cohort's day.

Leaving the decision to course instructors, however, does not ensure that each student is assigned to only one cohort. We developed a global model to assign all hybrid students to distinct cohorts.

The primary operational decision is determining which students to assign to each cohort. Randomly assigning students to cohorts resulted in some classrooms exceeding their social-distancing capacity on certain days. Therefore, for the 3CM, our objective is to determine a trisection of students that minimizes the total overage from one-third of the registered class size for *every* course meeting. For example, in a course with 20 students, we would want 7 students in two of the cohorts and 6 in the third. Because students register in different sets of courses, cohort assignment to course meetings can vary significantly. The total sizes of the three cohorts may differ so long as the students are assigned in a way that allows as many classrooms as possible to hold no more than one-third of their enrolled students during any given meeting.

We can visualize the model on a bipartite graph with a set of vertices representing hybrid students and a set representing courses, in which there is an edge from every student to each course that the student attends in person. Our goal is to partition the vertex set of students into three disjoint sets (cohorts) so that no course has significantly more than one-third of its students in any one cohort.

The *capacity* of each course is defined to be one-third of the course's enrollment because we developed this model before obtaining the list of social-distancing room capacities.

An *ideal* 3-partitioning of the students would be a partition such that no cohort in any course exceeds its capacity. Although it seems similar, we remark that this 3-partitioning is not a 3-coloring as normally defined on a graph (such as in Kelly and Kelly (1954)).

Figure 2 shows an example of an ideal 3-partitioning of the student vertices. Notice that each course's students are split as evenly as possible among the three cohorts. We cannot guarantee that an ideal 3-partitioning exists on the bipartite graph, so for such a graph, there may be at least one course in which the number of students in a cohort exceeds one-third of the course's enrollment. This surplus of students in each course is considered to be the *excess* of the corresponding course vertex. The sum of the excesses of the course vertices is the excess of the graph. Figure 3 shows an example of a nonideal 3-partitioning. In this figure, there is no way to choose cohorts for the four students so that each course has no more than one member from each cohort. For instance, if students 1 and 4 are assigned

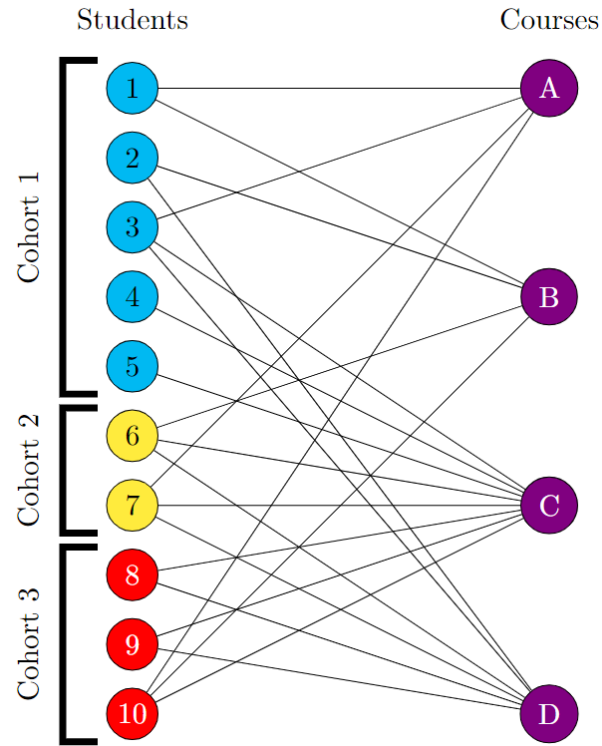


Figure 2. (Color online) This Graph Represents an Ideal 3-Partitioning of 10 Students and Four Courses: In Each Course, No More Than One-Third of Registered Students Are Assigned to Any Cohort

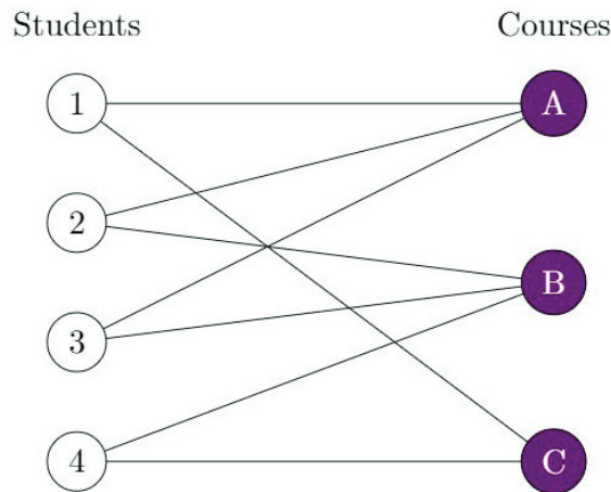


Figure 3. (Color online) This Graph Has No Ideal 3-Partitioning; There Is No Partitioning of the Students into Three Cohorts Such That Each Course Has No More Than One-Third of Its Enrollment in Each Cohort

to cohort 1, student 2 is assigned to cohort 2, and student 3 is assigned to cohort 3, then course C has two students in cohort 1 and therefore has an excess of one.

	Monday	Tuesday	Wednesday	Thursday	Friday
Aug.			19	20	21
	24	25	26	27	28
	31	1	2	3	4
Sept.	7	8	9	10	11
	14	15	16	17	18
	21	22	23	24	25
Oct.	28	29	30	1	2

Figure 4. This Calendar Shows Cohort Meeting Dates from August 19 to October 2: Distinct Cohorts Meet on Monday, Wednesday, and Friday Each Week and Rotate Through Tuesdays and Thursdays over a Three-Week Cycle
Note. Bold lines separate three-week cycles.

The 3CM is solved as a integer linear program that minimizes the total excess of the graph by choosing the cohorts for every student. This program is integer because we must use a binary decision variable for each student-cohort pair, where the variable is 1 if the student is assigned to the cohort and 0 otherwise. The constraints for the linear program are as follows:

- Each student must be assigned to exactly one cohort.
- The excess for each course is defined to be the largest cohort in the course minus the capacity of the course if it is positive and 0 otherwise.

A mathematical formulation of this model appears in Appendix B.

We ran this model on a laptop computer with 12 gigabytes of RAM and an Intel Core i5-8250U CPU 1.60 GHz processor under Windows 10. The problem contained 19,400 students across 1,718 courses. It loaded, solved to optimality, and generated output in proper form in less than 10 minutes, with the actual solving of the model taking less than 15 seconds. CPLEX Studio version 12.10 was used to solve the system in C++ using Visual Studio Community 2017 version 15.9.38.

As this model partitions Clemson students attending hybrid classes into three distinct cohorts, it creates three major concerns for the administration. First, this model requires the use of a cohort calendar, such as that shown in Figure 4. Each day is assigned a cohort, and only that cohort is able to attend in-person classes on that day. For any course that has a meeting schedule other than Monday-Wednesday-Friday, students would not be guaranteed an in-person course meeting each week. Second, some courses (such as those

that meet only once a week) were forced to be online-only for model simplicity and other practical concerns, such as the fact that students may get only one in-person meeting in a month. Third, once we received the actual room capacities for each classroom, optimal solutions of the 3CM that utilized this data produced cohorts that exceeded the designated capacity of the classroom by as much as 40%. Changing the objective to minimize the maximum course excess rather than the sum of course excesses produces smaller but more numerous capacity violations and does not necessarily produce a more desirable result. This problem could be overcome by reassigning courses with positive excess to larger classrooms to the extent possible, although the authors were not given the explicit freedom to change classrooms. Regardless, based on these concerns, the administration determined that the 3CM was unworkable for the immediate needs of Clemson University.

Although the 3CM approach did not fulfill Clemson's requirements, it may be well suited for situations in which all courses meet on the same set of days (such as a high school with daily classes) or settings in which course patterns can be changed to comply with a fixed daily pattern. The approach is easily generalized to work for arbitrary numbers of cohorts. We present our analysis of using the 3CM on Clemson University data in the Results section. To account for these concerns, we developed our second model, the Once-a-Week Model (OWM) described in the following section.

Once-a-Week Model

Because of the administrative concerns associated with the Three-Cohort Model, we next developed the Once-a-Week Model. The OWM uses a different approach to limit the number of student interactions. Rather than partitioning the entire student body into three distinct groups as the 3CM does, students are split into attendance groups for each course individually. In particular, the OWM assigns each student precisely one day of attendance per week for each course. We temporarily ignore classroom capacities in favor of creating roughly equal daily attendance per course meeting. We then develop an objective that minimizes student interactions across the university.

The constraints assigning each student to a single day on campus in each course can be visualized as a network flow model. The network is constructed with three layers of nodes. The first layer contains a node with supply 1 for each student-course pair (i.e., each student and each course that student attends in hybrid mode). The second layer contains a node for

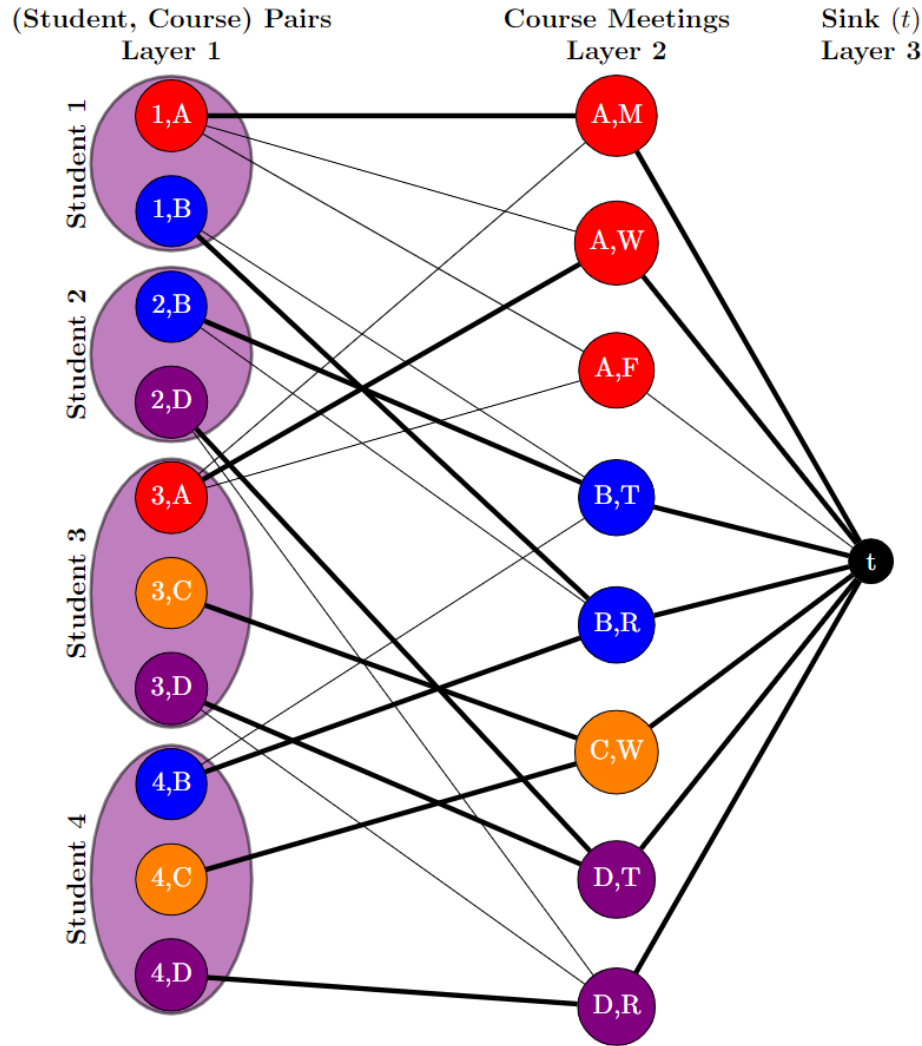


Figure 5. (Color online) This Three-Layer Network Depicts Four Students and Four Courses with Three, Two, One, and Two Weekly Meetings, Respectively

Notes. The bold lines represent a feasible flow. Each student-course node has a supply of 1; the sink has a demand equal to the number of student-course pairs, and arcs flow from left to right. Bold arcs between layers 1 and 2 carry a flow of one, and arcs between layer 2 and the sink have capacities equal to the meeting room capacity.

every meeting of every hybrid course. Uncapacitated arcs connect each student-course pair to each course-meeting pair for that course. The third layer is a global sink with demand equal to the total supply from the first layer. Arcs connect nodes in the second layer to the sink with capacity equal to the capacity of the meeting room. Thus, a feasible flow assigns every student to one meeting of each class that the student attends without violating the room capacity of any class meeting. An example of such a graph can be seen in Figure 5.

Choosing an Objective in the OWM

The network flow problem has many feasible solutions. So we now seek to pick a feasible flow that reduces student and faculty exposure risk. One way to accomplish this goal is to choose a feasible flow that minimizes interaction between students across the university.

Minimizing Interaction. In this formulation, we count an interaction between two students if they attend the same meeting of any class they share. A potential interaction can be modeled by a product of (integer-valued) flows on pairs of arcs between layer 1 and layer 2 that connect two students to the same course meeting node. The product is 1 if both students attend the same meeting of the shared course and 0 otherwise. If two students interact multiple times in different meetings, however, we count only a single interaction. We desire for the total number of interactions to be minimized.

This objective, together with the necessary constraints, can be used to place students in meetings. Because each potential interaction requires a product of two decision variables, our optimization problem includes quadratic constraints. This particular instance of the interaction-based model (more than 21,000 students) could not be solved in the time allocated by the administration.

Initial attempts to solve this model on a personal laptop failed when the model failed to build in a period of 6 hours. Attempts to solve this model on a Dell Precision Tower 7910 with 12 Intel Xeon E5-2643 cores and 78 GB RAM, with clock speed 3.4 GHz, with operating system CentOS Linux 8 compiling with gcc 8.3.1 and optimization solver Gurobi version 9.11 were unable to yield results better than an optimality gap of 70% after run times of more than 12 hours. This occurred even after reformulating the mixed-integer quadratic program into a mixed-integer linear program and using Gurobi's built-in Pre-QLinearize parameter set equal to 1. Therefore, we moved toward a proxy objective that would be more computationally efficient while still working to minimize interactions.

Minimizing Days per Week. Clearly, bringing students onto campus makes them open to more interactions than a model can account for. We cannot measure interactions based on where they choose to park, which walkways they use, where they spend time between classes, etc. Therefore, we decide that minimizing the number of days a week a student would need to come to campus may limit interactions by deterring students from visiting campus when they did not have an in-person meeting to attend.

We developed this component of the model by utilizing the network in Figure 5. We group arcs from layer 1 to layer 2 if they originate from the same student and terminate on the same day of the week. We want to find a minimum-cost feasible flow from layer 1 to layer 3 where a unit cost is incurred if any arc in a group carries positive flow. Therefore, for each day that a student attends classes in person, the graph imputes a fixed charge of 1 unit. For instance, in Figure 6, arcs $(4, B, T)$ and $(4, D, T)$ are grouped together as part of the K_{4T} set; if either has flow, then the network expresses a unit cost. We seek to minimize the cost across the entire network. A mathematical formulation of this model is detailed in Appendix C. For more information regarding fixed-charged networks, we direct readers to the work of Hirsch and Dantzig (1968).

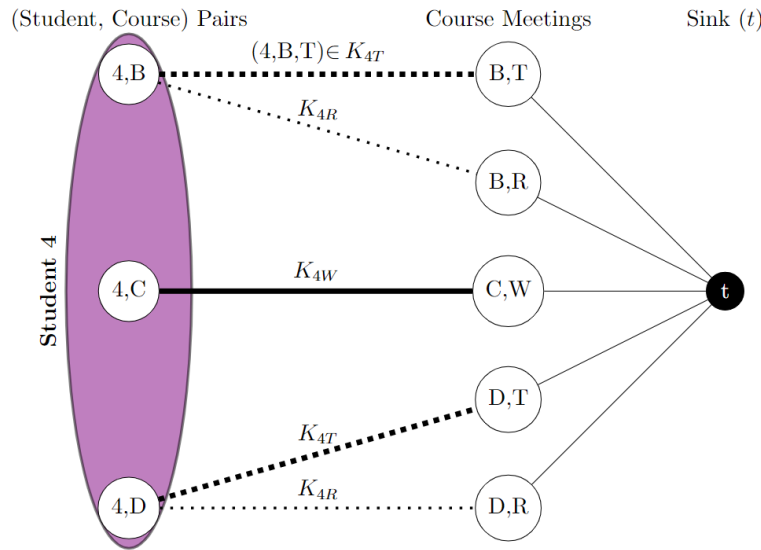


Figure 6. (Color online) This Subnetwork Depicts a Student from the Three-Layer Network with Three Courses
Note. The different styles of arcs from layer 1 to layer 2 denote distinct fixed-charge groups.

This configuration of the model results in a problem that can be handled computationally with fewer resources than the model minimizing interactions. Thus, we were able to run this model on a laptop with 12 GB of RAM and an Intel Core i5-8250U CPU 1.60 GHz processor under Windows 10. The problem contained 21,007 students across 3,059 courses. It was loaded, solved to optimality, and output in proper form in less than 11 minutes, with the actual solving of the model taking less than 30 seconds. CPLEX Studio version 12.10 was used to solve the system in C++ using Visual Studio Community 2017 version 15.9.38.

Results

In this section, we discuss the optimal solutions to each proposed model. We compare the 3CM and OWM with a default Round-Robin Strategy (RRS). The RRS operates similarly to the OWM by sequentially assigning each student a single day of the week to attend each course. However, the RRS makes no effort to minimize the number of student interactions or days attended on campus and is a proxy for the instructor-determined rotational approach. We also compare our models with a traditional scheduling of the fall 2020 semester. This comparison allows us to assess how well our implemented solution did compared to a default approach.

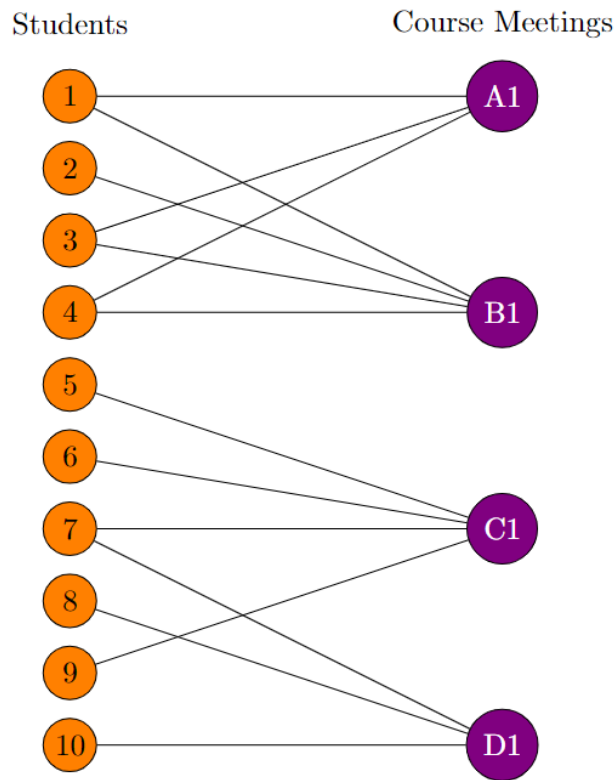


Figure 7. (Color online) This Graph Shows Student-Meeting Attendance for One Meeting Each of Four Different Courses
Note. The average number of days each student has an in-person meeting is 1.4, and there are 10 students with at least one in-person meeting.

We conceptualize the solutions via two graphs. Figure 7 depicts a bipartite student-meeting graph, in which an edge is present if a student (left node) attends a course meeting (right node) in person. Figure 8 is the student-interaction graph based on Figure 7. In the student-interaction graph, there is an edge between two students (nodes) if they share a

meeting. To assess the efficacy of our models, we introduce five performance measures that address concerns of Clemson University administration. The first three utilize common graph-theoretic concepts, and all five are rooted in the problem context. The performance measures are as follows:

- The average number of other students with whom each student shares a classroom, which is the average degree of each student-node in the student-meeting graph (Figure 7). The degree of each student-node is the number of potential exposures each student has during the week in their classes. A lower average value means that students share classrooms with fewer students.
- The partitioning of the student-interaction graph (Figure 8) provides disjoint sets of students who do not share any classrooms, even “transitively,” with students in other components. The more components a student-interaction graph has, the “better” the solution performs at reducing class-assignment interaction.
- The number of students in the three largest components of the student-interaction graph is a measure of how well the students are partitioned into even groups.
- Clemson University administrators believed that the fewer days students were on campus, the more likely informal meetings between students could be avoided, resulting in less infection spread. We report the average number of days per week that students are on campus.
- However, Clemson University administrators also wanted every student to be able to attend at least one class in person per week to provide more opportunity for engaged learning. We report the number of students who have at least one class in person per week.

Table 1 summarizes the performance measures for four scenarios. The first column (Traditional) represents the fall 2020 data for in-person students if courses had been held in their traditional manner. This measure gives us a base level for the performance metrics. The solution found by RRS can be considered the worst case of the hybrid assignment methods and is intended to simulate faculty assignments of days of attendance for their courses. The 3CM and OWM columns summarize results from the models presented.

As expected, RRS, 3CM, and OWM improve upon the fall 2020 base values for reducing the average student degree and the average days per week on campus while increasing the number of graph components. Although the 3CM creates three large components and creates more components than any other approach, 1,341 courses and 1,607 students

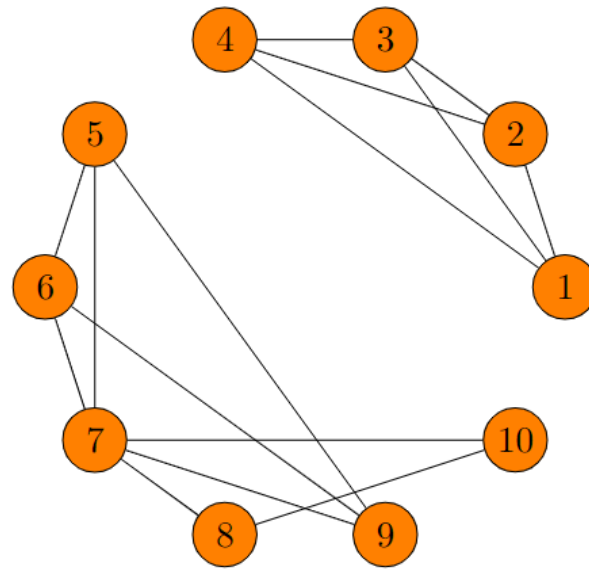


Figure 8. (Color online) This Student-Interaction Graph Is Derived from the Connections in Figure 7
Notes. Nodes represent students; edges pair students who attend the same course meeting. The average student degree is 3.0. There are two distinct components with six and four students respectively.

Table 1. This Table Displays Performance Measures for Fall 2020 Data for the Four Scenarios: Traditional, Round-Robin, 3CM, and OWM

	Traditional	RRS	3CM	OWM
Average student degree	587.28	84.44	56.98	82.70
Number of graph components	51	128	260	131
Number of students in three largest components	20,518	20,255	6,853	20,243
	64	57	6294	57
	57	44	5376	44
Average days/week students are on campus	3.92	2.64	1.59	1.94
Number of students attending in person	21,007	21,007	19,400	21,007

are forced to attend all classes online because their schedules include neither Monday-Wednesday-Friday nor Tuesday-Thursday classes. Therefore, 3CM is inappropriate if we desire to maximize the number of students attending some in-person classes. The notable difference between the RRS and OWM performance measures is that the OWM yields a lower average number of days per week on campus, which is expected because of its objective.

Table 2. This Table Displays the Number of Students on Campus Each Day Under Each Scenario

	Traditional	RRS	3CM	OWM
M	17,256	11,761	5,855	7,691
T	16,563	12,234	4,875	8,899
W	17,478	11,996	6,287	8,443
R	16,620	11,944	5,242	10,346
F	13,794	7,533	3,968	5,327
Alt T/R			4,535	

Table 2 reports the number of distinct students on campus each day based on each method. The number of students on campus on any given day varies in the traditional scenario because of the natural differences between course offering patterns (e.g., Friday afternoon classes are uncommon). All approaches reduce the number of students on campus each day, with the 3CM reducing the number of students the most. In the 3CM, we allow students to attend courses in person precisely one day per week of a Monday-Wednesday-Friday course and exactly twice in a three-week rotation of Tuesday1-Thursday1-Tuesday2-Thursday2-Tuesday3-Thursday3. Students attend each Tuesday-Thursday course twice in three weeks, and we denote the off-cycle day as “Alt T/R.” Comparing the daily patterns between RRS and OWM, the difference between these options becomes more apparent. The university implemented the OWM in fall 2020.

Discussion

Further Optimization by Minimizing Modality Clashes

Minimizing the total number of days that students come to campus is a relatively coarse objective that allows a significant number of optimal solutions, including simple permutations of schedules for small groups of students. In future work, we could consider a second-level optimization problem in which “modality clashes” are minimized among all minimum student-days solutions.

A modality clash occurs when a student has two or more courses on the same day for which their attendance modes (in person or online) are different. Figure 9 shows examples of modality clashes. The meetings that are bolded, underlined, and italicized indicate that the student is assigned to attend in person; plain text represents meetings that they should attend online. Modality clashes can be either acceptable or problematic. An acceptable

clash occurs if the classes that differ in mode are separated in time to reduce “slack” time on campus. The image on the right in Figure 9 is clearly a problematic modality clash because the student has back-to-back courses on Monday and Wednesday in different modalities. Our models did not address modality clashes, but we leave this avenue for future researchers to consider.

Day	8am	9am	10am	11am	12noon
M					HIST 1010.002
T		MATH 1150.002			
W		<u>BIOL</u> <u>1091.002</u>			HIST 1010.002
R		<u>MATH</u> <u>1150.002</u>			
F			<u>ED</u> <u>1050.005</u>		<u>HIST</u> <u>1010.002</u>

Day	8am	9am	10am	11am	12noon
M		AGM 4600.001	<u>PES</u> <u>2020.001</u>	AGM 4050.001	
T	HORT 4120.001				<u>HORT</u> <u>4121.001</u>
W		<u>AGM</u> <u>4600.001</u>	PES 2020.001	<u>AGM</u> <u>4050.001</u>	<u>PES</u> <u>2021.001</u>
R	<u>HORT</u> <u>4120.001</u>				<u>AGM</u> <u>4000.001</u>
F			PES 2020.001		

Figure 9. The Left Schedule Card Shows an Example of an Acceptable Modality Clash; the Schedule Card on the Right Shows an Example of a Problematic Modality Clash

Notes. (Left) Changes between in-person (bold underlined text) and online (regular format) modes are separated by free time. (Right) Monday and Wednesday schedules require switching between in-person and online modes with no attendant free block.

Registration Issues Occurring in Fall 2020

Registration of most university courses reflects a dynamic situation: students enroll late, change schedules, or drop courses; there is no date after which the registration lists are static. Students dropping courses can result in uneven daily attendance patterns, whereas new enrollments may induce interactions between sets of students that previously had no courses in common. In our application, new student registrations were handled in a greedy fashion by assigning each student’s in-person day for each course to the day that had the smallest current enrollment, regardless of their other course enrollments or in-person days; this approach did not consider the impact of these late-breaking course additions on the system performance measures.

Practical Considerations

In addition to handling changing data (immutable when considered, but with our knowledge that the data were changing dynamically), we discovered several additional nuances:

- At Clemson, 4000-level courses often have a 6000-level component. Such a course is considered “cross-listed.” Imagine a cross-listed course that meets twice a week, with 30 students in the 4000 level and 3 students in the 6000 level. Is it better to have all the 6000-level students attend on the same day, or is it better for one day to have one of the 6000-level students and the other day to have two? In our model, we aggregated the students in the cross-listed courses together and treated them without differentiation.

- At Clemson, there are multiple courses with two or more meetings on the same day. For example, there is a section of one calculus course that meets in two consecutive 75-minute class periods on Thursdays, and some construction management courses met twice on Fridays. These courses required additional variables within our model to separately handle which course meeting a student attends and which day they attend that course. This added variables and constraints to the model on the order of the total number of student-course pairs.

- Some courses have a teamwork component, especially capstone courses in the engineering programs. Enrollment in those courses tends to be stable because students often take them in the last semester of their undergraduate degrees. Instructors would like to assign students to teams before the semester begins, creating effective teams for the design problems to be considered. However, the university delayed the start of in-person instruction and had one month of online education. To accommodate this delay, days of attendance were assigned after late registration so that the most accurate data were available. The fact that some teams did not have common days of attendance frustrated instructors and probably impacted the student experience in these courses.

- No one knows individual curricula better than the department faculty and relevant staff. Courses are often scheduled in ways that facilitate student learning. For example, a course may have a lecture and laboratory component scheduled with different daily patterns (such as Monday and Wednesday for the lecture and Tuesday and Thursday for the laboratory component). The instructor may want the students attending Monday’s lecture in person to take the Tuesday laboratory section. Our methodology does not address this concern.

- Student course selection was considered inviolable. If a student registered for a course, our solution had to allow the student to attend the course, even if moving the student to a course with equivalent content and different meeting times would benefit the performance measures.

- After conducting the fall 2020 semester, there is anecdotal evidence to suggest many students who elected hybrid mode chose not to attend their classes in person. That choice may have been due to modality clashes and was also possibly affected by Clemson's all-online start to the semester.

Informing the Students and Instructors

Clemson University utilized the results of the OWM model and disseminated that information to faculty and staff through the COVID-19 section of `my.Clemson.edu`, using the process in Figure 10. The fixed static output was provided to the course instructors via a website, with a view such as Figure 11. Students were informed of their attendance modes via an app, with images such as Figure 12. Every student's weekly pattern was fixed for the entire semester and disseminated before the beginning of each semester's course offerings.

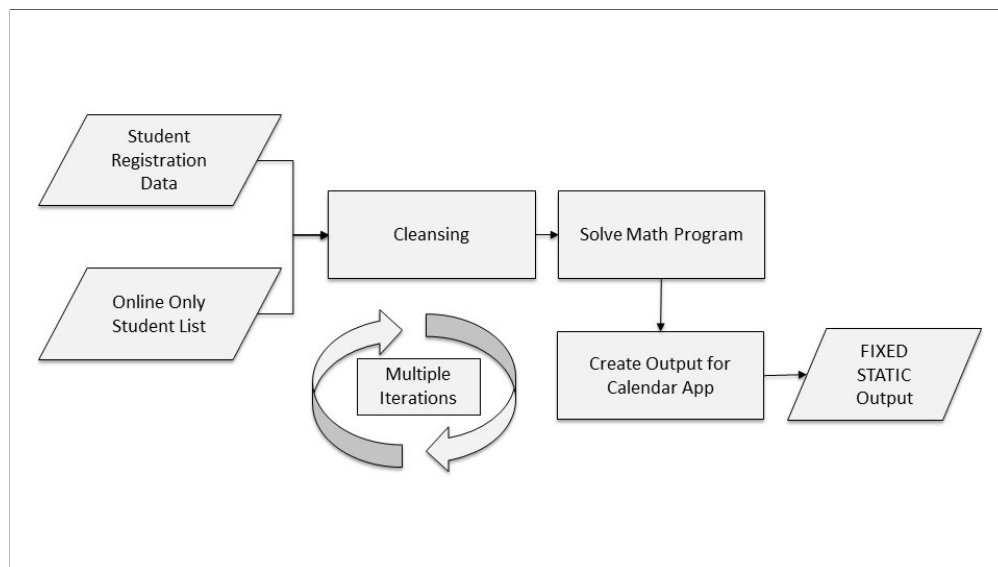
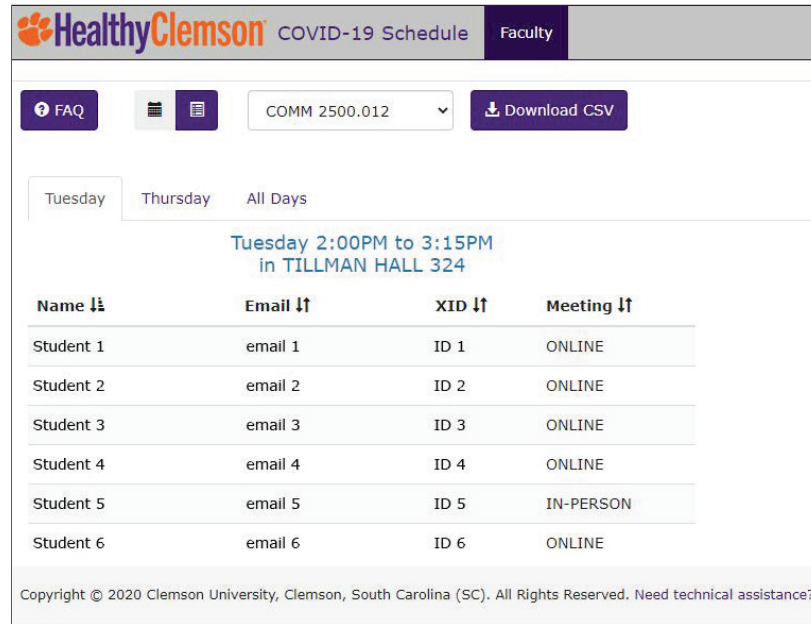


Figure 10. This Flowchart Depicts an Overview of the Stages and Flow of Project Implementation

Conclusions

A team of researchers at Clemson University implemented a rotational attendance policy in which the institution assigned students to attendance days. The approach used an integer program based on enrollment data. The daily attendance data were provided to instructors and students via an app or website, with late-breaking additions being determined daily in a deterministic greedy manner. A hypothetical instructor-determined attendance pattern was modeled as a round-robin attendance pattern. Both OWM and RRS reduce the average



HealthyClemson COVID-19 Schedule Faculty

FAQ COMM 2500.012 Download CSV

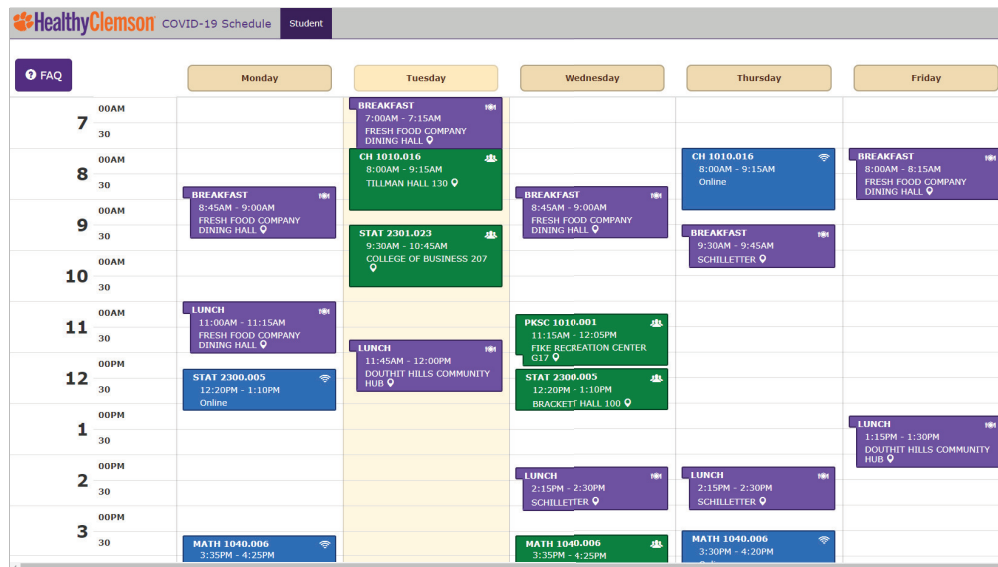
Tuesday Thursday All Days

Tuesday 2:00PM to 3:15PM in TILLMAN HALL 324

Name	Email	XID	Meeting
Student 1	email 1	ID 1	ONLINE
Student 2	email 2	ID 2	ONLINE
Student 3	email 3	ID 3	ONLINE
Student 4	email 4	ID 4	ONLINE
Student 5	email 5	ID 5	IN-PERSON
Student 6	email 6	ID 6	ONLINE

Copyright © 2020 Clemson University, Clemson, South Carolina (SC). All Rights Reserved. Need technical assistance?

Figure 11. (Color online) This Web Page Shows the Instructor's View of a Sample Rotational Attendance Plan
Source. Upp (2021).



HealthyClemson COVID-19 Schedule Student

FAQ

	Monday	Tuesday	Wednesday	Thursday	Friday
7:00AM		BREAKFAST 7:00AM - 7:15AM FRESH FOOD COMPANY DINING HALL			
8:00AM		CH 1010.016 8:00AM - 9:15AM TILLMAN HALL 130		CH 1010.016 8:00AM - 9:15AM Online	BREAKFAST 8:00AM - 8:15AM FRESH FOOD COMPANY DINING HALL
9:00AM	BREAKFAST 8:45AM - 9:00AM FRESH FOOD COMPANY DINING HALL	STAT 2301.023 9:30AM - 10:45AM COLLEGE OF BUSINESS 207	BREAKFAST 8:45AM - 9:00AM FRESH FOOD COMPANY DINING HALL	BREAKFAST 9:30AM - 9:45AM SCHILLETTER	
10:00AM					
11:00AM	LUNCH 11:00AM - 11:15AM FRESH FOOD COMPANY DINING HALL	LUNCH 11:45AM - 12:00PM DOUTHITT HILLS COMMUNITY HUB	PKSC 1010.001 11:15AM - 12:05PM FIKE RECREATION CENTER G17		
12:00PM	STAT 2300.005 12:20PM - 1:10PM Online		STAT 2300.005 12:20PM - 1:10PM BRACKETT HALL 100		
1:00PM					LUNCH 1:15PM - 1:30PM DOUTHITT HILLS COMMUNITY HUB
2:00PM			LUNCH 2:15PM - 2:30PM SCHILLETTER	LUNCH 2:15PM - 2:30PM SCHILLETTER	
3:00PM	MATH 1040.006 3:35PM - 4:25PM		MATH 1040.006 3:35PM - 4:25PM	MATH 1040.006 3:30PM - 4:20PM	

Figure 12. (Color online) This Web Page Shows a Student's View of a Sample Daily Attendance Mode Schedule
Source. Upp (2021).

number of weekly student interactions from about 587 (in a pre-COVID-19 schedule) to ~ 84 (RRS) or ~ 83 (OWM). By implementing the OWM model as opposed to RRS, Clemson University was also able to reduce the average number of students on campus from $\sim 7,500$ – $12,200$ (RRS) to $\sim 5,300$ – $10,300$ (OWM), further reducing the potential of

interaction outside of class while on campus. The project was deemed successful by the administration and was also used to determine hybrid attendance of students in the spring 2021 semester for those classes still maintaining a hybrid modality.

Upon reflection, it is not clear that a rotational attendance policy would be implemented at Clemson if such an unfortunate scenario were to occur again. The intention was to have students in person on a periodic basis, assuming that students would actually come to campus. In reality, the modality clash impacted student attendance, but so did human nature (in fall 2020) and pandemic fatigue (in spring 2021). Faculty began teaching in person in their hybrid classes, but with university-encouraged relaxed attendance policies, student attendance declined and in some classes, faculty would arrive to empty rooms. During summer 2020, leaders in Student Affairs and Undergraduate Studies were faced with parent feedback about continued online instruction and wanted a way to allow as many students on campus as possible while reducing the risk of transmission based on classroom interaction. The administration-imposed requirements that students attend their hybrid courses exactly once a week and experience no schedule changes precluded either a timetabling approach or a solution in which some courses were forced to be online only and others were allowed (or even forced) to be in person. The technical challenges to creating a rotational attendance pattern are easily addressed, as evidenced by this paper, whereas the human behavior issues are neither easily addressable with a technical solution nor easily foreseeable in the new pandemic environment. Archiving the data used for this study, currently underway, will allow future research by interdisciplinary teams of optimizers and student affairs professionals to better address this problem subject to administrative constraints.

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- Robert Jones, Executive Vice President for Academic Affairs and Provost

A. Notation Used in Mathematical Models

Sets

Symbol	Definition
S	Hybrid students
K	Hybrid courses
S_k	Hybrid students enrolled in hybrid course k
K_s	Hybrid courses that student s is enrolled in
D	Days of the week that the University holds classes
D_k	Days of the week on which course k meets
C	The set of cohorts

Three-Cohort Model

Symbol	Definition
γ_k	<u>Input parameter</u> Student capacity of any meeting of course k
n_{kc}	<u>Decision variable</u> Number of hybrid students in cohort c that attend course k in person
x_{sc}	<u>Decision variable</u> 1 if student s is assigned to cohort c ; 0 otherwise
α_k	<u>Decision variable</u> Maximum capacity violation for any meeting of course k

Once-a-Week Model

Symbol	Definition
$\rho_{ss'}$	<u>Decision variable</u> 1 if students s and s' share any in-person course meeting; 0 otherwise
y_{skd}	<u>Decision variable</u> 1 if student s attends course k in person on day d ; 0 otherwise
v_{kd}	<u>Decision variable</u> Number of students attending course k in person on day d
w_{sd}	<u>Decision variable</u> 1 if student s attends any course in person on day d ; 0 otherwise

B. Three-Cohort Model

$$\text{minimize } \sum_{k \in K} \alpha_k \tag{B.1}$$

$$\text{subject to } \sum_{c \in C} x_{sc} = 1 \quad \forall s \in S \tag{B.2}$$

$$\sum_{s \in S_k} x_{sc} = n_{kc} \quad \forall c \in C, \forall k \in K \tag{B.3}$$

$$\alpha_k \geq n_{kc} - \gamma_k \quad \forall c \in C, \forall k \in K \tag{B.4}$$

$$\alpha_k \geq 0 \quad \forall k \in K \tag{B.5}$$

$$x_{sc} \in \{0, 1\} \quad \forall s \in S, \forall c \in C \tag{B.6}$$

The 3CM is a mixed-integer linear program derived from a bipartite graph such as in Figure 2 that attempts to find a 3-partitioning of the student set of vertices. The feasible set of this model is constructed by constraints (B.2), (B.3), and (B.4).

- Equation (B.2) requires that each student be assigned to exactly one cohort.
- Equation (B.3) defines the variable n_{kc} to be the assigned attendance for the course k and cohort c .
- Inequality (B.4), together with objective (B.1) defines α_k to be the maximum capacity violation for course k across all cohorts.
- The objective in Equation (B.1) accumulates the maximum capacity violation for each course and attempts to minimize this total.

C. Once-a-Week Model

Minimizing interaction:

$$\text{minimize } \sum_{s \in S} \sum_{s' \in S \setminus \{s\}} \rho_{ss'} \quad (\text{C.1})$$

$$\text{subject to } \sum_{d \in D_k} y_{skd} = 1 \quad \forall s \in S, \forall k \in K_s \quad (\text{C.2})$$

$$\sum_{s \in S_k} y_{skd} = v_{kd} \quad \forall k \in K, \forall d \in D_k \quad (\text{C.3})$$

$$0 \leq v_{kd} \leq \lceil |S_k| / |D_k| \rceil \quad \forall k \in K, \forall d \in D_k \quad (\text{C.4})$$

$$y_{skd} \cdot y_{s'kd} \leq \rho_{ss'} \quad \forall s, s' \in S, \forall k \in K_s \cap K_{s'}, \forall d \in D_k \quad (\text{C.5})$$

$$y_{skd} \in \{0, 1\} \quad \forall s \in S, \forall k \in K, \forall d \in D_k \quad (\text{C.6})$$

$$\rho_{ss'} \in \{0, 1\} \quad \forall s, s' \in S \quad (\text{C.7})$$

Minimizing days per week in person:

$$\text{minimize } \sum_{s \in S} \sum_{d \in D} w_{sd} \quad (\text{C.8})$$

subject to (C.2), (C.3), (C.4), (C.6), and

$$y_{skd} \leq w_{sd} \quad \forall s \in S, \forall k \in K, \forall d \in D_k \quad (\text{C.9})$$

$$w_{sd} \in \{0, 1\} \quad \forall s \in S, \forall d \in D \quad (\text{C.10})$$

The network flow model discussed in the paper can be visualized as Figure 5. The feasible set of this model is constructed by constraints (C.2), (C.3), (C.4), and (C.6).

- Equation (C.2) requires that each student s be assigned to exactly one meeting for each course for which they are enrolled. This represents the supply in layer 1 of Figure 5.
- Equation (C.3) balances the flow into and out of layer 2. This also defines the variable v_{kd} , the flow variable for arcs connecting layers 2 and 3.
- Inequality (C.4) restricts the number of students in each meeting, the flow v_{kd} , to be nonnegative and less than the number of students divided by meetings. This allows for a close to equal number of students assigned to each meeting. This is the capacity constraint on arcs between layers 2 and 3.

The two possible adaptations of the OWM, minimizing interaction and minimizing days per week, are each formed by an additional constraint equation and an objective function.

- Equation (C.5) defines the binary variable $\rho_{s_1 s_2}$ to represent when students s_1 and s_2 interact.
- The objective in Equation (C.1) minimizes the sum of all interactions across all possible student pairs.
- Equation (C.9) represents the fixed charge in Figure 6 and defines the binary variable w_{sd} to represent a student s attending at least one meeting in person on day d .
- The objective in Equation (C.8) minimizes the number of days per week a student attends a meeting in person.

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Amy B. Gore is a process improvement engineer. She received her master’s degree in mathematical sciences from Clemson University in 2020 focusing on operations research. Her professional interests include leveraging data analysis and optimization models for business decisions.

Mary E. Kurz is an associate professor of industrial engineering in the College of Engineering, Computing and Applied Sciences at Clemson University. Her research interests lie in the area of tactical decisions for assembly systems, especially in line balancing, scheduling, and sequencing.

Matthew J. Saltzman is associate professor of mathematical and statistical sciences at Clemson University. His research interests are in computational optimization. He is the founding president of the COIN-OR Foundation, an educational nonprofit publisher of open-source software for operations research.

Blake Splitter is a PhD student studying operations research at Clemson University. His primary research interests include metaheuristics and quadratic programming.

William C. Bridges is a professor in the School of Mathematical and Statistical Sciences at Clemson University. His primary professional interests involve collaboration with colleagues across the university on statistical and mathematical aspects of their research.

Neil J. Calkin is a professor in the School of Mathematical and Statistical Sciences at Clemson University. He earned a PhD from the Department of Combinatorics and Optimization at the University of Waterloo in 1988.